DUSO Mathematics League 2016 - 2017

CONTEST #1.

SOLUTIONS

1 - 1. $\boxed{\mathbf{100}}$ The inequality can be solved to obtain $20x \leq 2000 \to x \leq 100$. Thus, the answer is **100**.

1 - 2. $\boxed{\{\mathbf{-1, 2}\}\ \textit{need both}}$ This equation is of the form $A^2 + B^2 = (A+B)^2$, which has solutions only if $A = 0$ or $B = 0$. Therefore, instead of expanding the brackets and proceeding to solve a quadratic equation, instead solve two linear equations to find $x + 1 = 0 \to x = -1$ and $x - 2 = 0 \to x = 2$. The solutions are $\{\mathbf{-1, 2}\}$.

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1 - 3. $\boxed{\mathbf{7}}$ Suppose the length of one of the congruent sides is 3 and the non-congruent side has length 1 (which is minimal). In that case, the perimeter is $3 + 3 + 1 = 7$. If 3 is the length of the non-congruent side, then the minimum perimeter occurs if the congruent sides measure 2 (notice that a 1-1-3 triangle does not exist). The perimeter in this case is $2 + 2 + 3 = 7$. In either case, the perimeter is **7**.

1 - 4. $\boxed{\mathbf{84}}$ Notice first that $\triangle EUR \sim \triangle ESA$ and the sides of the triangles are in the ratio $1 : 2$, so the area of $\triangle EAS$ is $2^2 \cdot 7 = 28$. Now, notice that E is equidistant from $\overline{AU}$ and $\overline{UR}$, so those two triangles have areas in the same ratio as their bases, and $AU : UR = 2 : 1 \to$the area of $\triangle AEU$ is $2 \cdot 7 = 14$. Because $\triangle SAU$ has area $28 + 14 = 42$, the area of square is $42 \cdot 2 = \mathbf{84}$.

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1 - 5. $\boxed{\mathbf{7}}$ The remainder when dividing by $x - 4$ is the same as the function evaluated at 4, so the remainder is $4^3 - 4 \cdot 4^2 + 12 - 5 = \mathbf{7}$.

1 - 6. $\boxed{\frac{\mathbf{3}}{\mathbf{2}}}$ The roots are $q - d$, q, and $q + d$, so $r - p = 2d$ for the difference d of the arithmetic progression. From Viete's formulas, we have the sum of the roots of this cubic equation is $\frac{3}{2}$, so the root q is $\frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$. The product of the roots is $\frac{-5}{32}$ by Viete's formulas, and this product is $q(q^2 - d^2)$, so solve $\frac{1}{2}\left(\frac{1}{4} - d^2\right) = \frac{-5}{32}$ to obtain $d = \frac{3}{4}$. Therefore, $r - p = 2d = \frac{\mathbf{3}}{\mathbf{2}}$.

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R-1. If 12% of a number is 144, compute the number.

R-1Sol. $\boxed{\mathbf{1200}}$ Solving $\dfrac{144}{N} = \dfrac{12}{100}$ obtains $12N = 14400 \rightarrow N = 1200$.

R-2. *Let N be the number you will receive.* If $N = A \cdot B!$ for some positive integers A and B, compute the least possible value of A.

R-2Sol. $\boxed{\mathbf{10}}$ To minimize A, maximize B. To maximize B, look for the greatest factorial that divides N. Substituting, we see that $1200 = 10 \cdot 5!$, so $A = \mathbf{10}$.

R-3. *Let N be the number you will receive.* When the hands of a standard clock are at N o'clock, compute the measure of the supplement of the acute angle between the hands.

R-3Sol. $\boxed{\mathbf{120}}$ Substituting, at $10:00$, the hands are separated by $1/3$ of $180°$, or 60 degrees. The supplement measures $180 - 60$ or **120** degrees.

R-4. *Let N be the number you will receive.* Compute the least positive integer x such that $\sqrt{2N + x^2}$ is a whole number.

3-4Sol. $\boxed{\mathbf{4}}$ Substituting, look for x such that $\sqrt{240 + x^2}$ is a whole number. The least x that satisfies the conditions of the problem is $x = \mathbf{4}$, in which case $\sqrt{256} = 16$ is a whole number.

R-5. *Let N be the number you will receive.* A set of N consecutive whole numbers has a sum of 2018. Compute the greatest of the whole numbers.

R-5Sol. $\boxed{\mathbf{506}}$ Substituting, there are 4 consecutive whole numbers, whose sum is $x + x - 1 + x - 2 + x - 3 = 4x - 6 = 2018$. Solving, $x = \mathbf{506}$.